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Light-like noncommutativity and duality from open strings/branes

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Abstract: In this paper we perform some non-trivial tests for the recently obtained open membrane/D-brane metrics and 'generalized' noncommutativity parameters using Dp/NS5/M5-branes which have been deformed by light-like fields. The results obtained give further evidence that these open membrane/D-brane metrics and 'generalized' noncommutativity parameters are correct. Further, we use the open brane data and supergravity duals to obtain more information about non-gravitational theories with light-like noncommutativity, or 'generalized' light-like noncommutativity. In particular, we investigate various duality relations (strong coupling limits). In the light-like case we also comment on the relation between open membrane data (open membrane metric etc.) in six dimensions and open string data in five dimensions. Finally, we investigate the strong coupling limit (high energy limit) of five dimensional NCYM with $\Theta^{12} = \Theta^{34}$. In particular, we find that this NCYM theory can be UV completed by a DLCQ compactification of M-theory.

Keywords: p-branes, String Duality, AdS-CFT Correspondence, Non-Commutative Geometry.

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Contents

1. Introduction

During the last couple of years there have been a huge interest in non-gravitational theories which have a noncommutative structure. The first so called noncommutative theory to be obtained was noncommutative super Yang-Mills (NCYM), with space-space [1]–[10] or light-like [11]–[14] noncommutativity. This was later followed by noncommutative open string theory (NCOS) [15]–[22], as well as various world volume theories (of deformed D-branes, NS5-branes and M5-branes) containing light open M2-branes or open D-branes, OM/ODp/$\widetilde{\mathrm{OD}q}$ [23]–[39] and [12].

It is well known that the open string metric, coupling constant and noncommutativity parameter [3], give important information for both NCYM and NCOS (e.g. in the mass shell condition for NCOS and in the definition of the Yang-Mills coupling constant for NCYM). These open string quantities are well defined and obtained from the open string two-point function [3] (metric and noncommutativity parameter) and the Dirac-Born-Infeld action (coupling constant, see e.g. [3]). For OM-theory it was conjectured in [24] that there is an open membrane (M2) metric which is relevant for this theory. Later in [32, 33, 38], the full open membrane metric was obtained, including the conformal factor, using different methods. Unfortunately we still lack a microscopic understanding of the open membrane metric. In [33, 38], the full structure of a conjectured open membrane generalized noncommutativity parameter was also obtained, again using different methods. This object is expected to be the open membrane analog of the open string noncommutative parameter, see [33, 38, 37] where the relation to the open string noncommutativity parameter is derived. In paper [33] expressions for open Dq-brane metrics and generalized noncommutativity parameters were also obtained. These are expected to be important for the open D-brane theories ODp [23, 26, 12, 27, 29, 30, 31] (NS5-brane world volume theories containing light open Dp-branes) and $\widetilde{\mathrm{ODq}}$ [29, 30] (D$(q+2)$-brane world volume theories containing light open Dq-branes).

In this paper we will further investigate these open brane (Dq/M2) metrics for the special case where we have turned on a light-like field on a Dp/NS5/M5-brane. In particular, we show that if one insert a supergravity solution, which corresponds to a Dp/NS5/M5-brane with a light-like field turned on, in these open metrics, one obtains a deformation independent [9] open brane metric.[1] We also show that one obtains the expected results after taking the near horizon limit (e.g. fixed generalized noncommutativity parameter in all cases). For a deformation with a magnetic or electric RR field, deformation independence and fixed generalized noncommutativity, was derived in [33]. Further, we find here that the open membrane metric and generalized noncommutativity parameter in six dimensions can be reduced to the open string metric, coupling constant and noncommutativity parameter in five dimension, in the case when the M-theory three form A is light-like (light-like $A \to$ light-like NS-NS B, where B is dual to a light-like RR three form C). In [33, 38], it was demonstrated that the reduction for a general three form only works for the 'electric' rank 2 case, see [33, 38] for details. However, the light-like case was excluded from the investigation. Further, we investigate field theories and Little String theories with light-light noncommutativity [11, 12], specially their duality relations. We also include a discussion about the strong coupling limit of five dimensional NCYM with $\Theta^{12} = \Theta^{34} \neq 0$, which is given by a DLCQ compactification of M-theory in the presence of a 'parallel' M5-brane (i.e., one of the M5-brane directions is in the light-like compactified direction) and two transverse M2-branes.

The paper is organized as follows: section 2 contains the relevant supergravity duals, as well as the open F1/Dq/M2 data, that are needed later. In section 3 we discuss various issues about field theories and Little String theories with light-like 'ordinary' or 'generalized' noncommutativity, as well as how open string and open membrane data are related in five and six dimensions. In section 4 we investigate the strong coupling limit of NCYM with $\Theta^{12} = \Theta^{34} \neq 0$ in five dimensions. There are also two appendices included, where we have collected our conventions regarding T-duality, S-duality and lift from type IIA to eleven dimensions, as well as a discussion on the relation between the 'supergravity dual' and the 'flat space scaling' approaches to noncommutative theories.

[1] This means that the result is independent of the deformation parameter θ in the solution, see section 2 and [40, 10, 33]. This result is important, since the assumption of deformation independence played an important role in the derivation of the open membrane/D-brane metrics in [33].

2. Supergravity duals and open brane data

In this section, we obtain the supergravity duals of all non-gravitational theories with a one parameter light-like noncommutativity, in dimensions $d \leq 6$.[2] The relevant open string/open D-brane data are also computed and shown to be deformation independent.

The supergravity duals are obtained by starting with the relevant supergravity solution, corresponding to a bound state (stack of N branes at $r = 0$ with different fields turned on), followed by taking a near horizon limit (involves $\alpha' \to 0$, see below). The so obtained supergravity solution is called a supergravity dual. This supergravity dual is then probed by a probe brane. By taking this probe brane infinitely far away from the stack (i.e., $\tilde{r} \to \infty$), the U(1) degrees of freedom on the probe brane will decouple from gravity, and the physics of the probe brane can be described by a non-gravitational theory with noncommutativity, see e.g. [9, 33, 41] for further clarifications. If e.g. the probe brane is a Dp-brane, then open strings can end on it. Therefore, the open string metric, coupling constant and noncommutativity parameter give important information of the physics on this probe brane. For example, as we will see below, in the case of a Dp-brane with a light-like B-field turned on, we obtain, using the supergravity dual and open string data, that the physics on the single probe brane is governed by a U(1) $(p+1)$-dimensional NCYM theory, with light-like noncommutativity $\Theta^{-1} \neq 0$.[3]

We note here, that this supergravity set up is equivalent to instead starting with flat space and then taking a decoupling limit a la e.g. [3, 23] in order to decouple gravity. In appendix B there is a further discussion on these two approaches and their equivalence. The main reason why we use the supergravity dual approach is because it gives us the opportunity to check that the recently derived open M2/Dq-brane metrics [32, 33] are deformation independent [9, 33], in the case of light-like deformations. That the open M2/Dq-brane metrics are deformation independent, implies that if one insert a deformed brane solution (see below) in the relevant open M2/Dq-brane metrics, then the result is not dependent of the deformation parameter θ, see [9, 10, 33] and below for details. Note also that it leads to fixed generalized noncommutativity parameters. This is important to check for light-like deformations, since the assumption of deformation independence played an important role when these open brane metrics were constructed in [33]. To be more precise: In [33] it was assumed that a Dp-brane solution deformed with an electric RR $(p-1)$ form, should give a deformation independent open D$(p-2)$-brane metric and fixed generalized noncommutativity parameter. This assumption was essential in the derivation of the complete open Dq-brane ($q = p - 2$) metric and generalized noncommutativity parameter, valid for a one parameter deformation (see [33] for more details). The M-theory open membrane data were derived using similar assumptions. Now, since these open D-brane metrics and generalized noncommutativity parameters should be deformation

[2]These supergravity duals have earlier been obtained in [12] (for the deformed Dp-brane cases, see also [13]), using a different solution generating technique. We will here obtain equivalent solutions, using the $\mathrm{O}(p+1, p+1)$ method [9, 10, 21], see below.

[3]For a stack of k probe Dp-branes on top of each other, we instead have a U(k) theory, where $k \ll N$, in order for the probe branes to have a negligible effect on the background. In this paper we usually set $k = 1$.

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independent for any one parameter deformation. It is important to show that this is indeed true. In [33], this was shown for all electric and magnetic deformations. Below we will show that also deformations with light-like RR fields give deformation independent results.

2.1 Deforming a Dp-brane with light-like NS-NS B-field

We start by deforming a Dp-brane with a light-like NS-NS B-field (i.e, $B_{+1} \neq 0$). To obtain this supergravity solution we use the $O(p+1,p+1)$ solution generating technique [9, 10, 21]. Using this method gives the following solution[4]

$$\begin{aligned}
ds^2 &= H^{-1/2}(2dx^- dx^+ - \theta^2 H^{-1}(dx^+)^2 + (dx^1)^2 + (dx^3)^2 + \cdots + (dx^p)^2) + \\
&\quad + H^{1/2}(dr^2 + r^2 d\Omega_{8-p}^2)\,, \\
e^{2\phi} &= g^2 H^{\frac{3-p}{2}}\,, \qquad B_{+1} = -\theta H^{-1}\,, \\
C_{+3\ldots p} &= g^{-1}\theta H^{-1}\,, \qquad H = 1 + \frac{Ng(\alpha')^{\frac{7-p}{2}}}{r^{7-p}}\,,
\end{aligned} \tag{2.1}$$

where the light-like coordinates $x^{\pm} = \frac{1}{\sqrt{2}}(x^2 \pm x^0)$. From a bound state point of view, this supergravity solution corresponds to a Dp-D$(p-2)$-F1-W bound state, where W means that there is a wave included in the bound state. To obtain the supergravity duals of various noncommutative theories (NCYM) we take the following 'magnetic' near horizon limit (similar to (5) in [30]), where

$$x^{\mu}\,, \qquad \tilde{r} = \frac{\tilde{\ell}^2}{\alpha'} r\,, \qquad \tilde{g} = g\Big(\frac{\alpha'}{\tilde{\ell}^2}\Big)^{\frac{p-3}{2}}\,, \qquad \tilde{\ell}^2 = \theta\alpha'\,, \tag{2.2}$$

are kept fixed in the $\alpha' \to 0$ limit. Inserting this limit in (2.1) gives the following supergravity duals

$$\begin{aligned}
\frac{ds^2}{\alpha'} &= \frac{1}{\tilde{\ell}^2}\Big(\frac{\tilde{r}}{\tilde{R}}\Big)^{\frac{7-p}{2}}\Big[2dx^- dx^+ - \Big(\frac{\tilde{r}}{\tilde{R}}\Big)^{7-p}(dx^+)^2 + (dx^1)^2 + (dx^3)^2 + \cdots + (dx^p)^2\Big] + \\
&\quad + \frac{1}{\tilde{\ell}^2}\Big(\frac{\tilde{R}}{\tilde{r}}\Big)^{\frac{7-p}{2}}(d\tilde{r}^2 + \tilde{r}^2 d\Omega_{8-p}^2)\,, \\
e^{2\phi} &= \tilde{g}^2\Big(\frac{\tilde{R}}{\tilde{r}}\Big)^{\frac{(7-p)(3-p)}{2}}\,, \qquad \frac{B_{+1}}{\alpha'} = -\frac{1}{\tilde{\ell}^2}\Big(\frac{\tilde{r}}{\tilde{R}}\Big)^{7-p}\,, \\
\frac{C_{+3\ldots p}}{(\alpha')^{\frac{p-1}{2}}} &= \frac{1}{\tilde{g}\tilde{\ell}^{p-1}}\Big(\frac{\tilde{r}}{\tilde{R}}\Big)^{7-p}\,, \qquad \tilde{R}^{7-p} = \tilde{g}N\tilde{\ell}^{7-p}\,.
\end{aligned} \tag{2.3}$$

Next, we calculate the open string metric, coupling constant and noncommutativity parameter, for a single Dp-brane probing this background solution at a distance $\tilde{r}$ from the stack. For a general background these are given by [3] (we use the same conventions as

[4] To be more specific, we use the formulas in [21, section 2] and deform the Dp-brane with $\theta^{01} = \theta^{12} = -\theta/\sqrt{2}$. Note that we only include the parts of the solution which are relevant in the investigation below, i.e., we do not include the RR $(p+1)$-form (which, anyway, is the same as for an undeformed Dp-brane).

in [30]):

$$
\begin{aligned}
G_{\mu\nu} &= g_{\mu\nu} + B_{\rho\mu} g^{\rho\sigma} B_{\sigma\nu} = g_{\mu\nu} + B^2_{\mu\nu} \,, \\
\Theta^{\mu\nu} &= -\alpha' g^{\mu\rho} B_{\rho\sigma} G^{\sigma\nu} \,, \\
G^2_{\rm os} &= e^{\phi} \left(\frac{\det G}{\det g} \right)^{1/4} \,.
\end{aligned} \tag{2.4}
$$

The symmetric tensor $G_{\mu\nu}$ is the open string metric governing the mass-shell condition for the open string states propagating on the probe D-brane, while the antisymmetric tensor $\Theta^{\mu\nu}$ is the parameter of noncommutativity between the D-brane coordinates. Inserting the above solution (2.1) in (2.4), gives the following deformation independent result (before taking the limit (2.2)):[5]

$$
G_{\mu\nu} = H^{-1/2} \eta_{\mu\nu} \,, \quad G^2_{\rm os} = g H^{\frac{3-p}{4}} \,, \quad \Theta^{-1} = \alpha' \theta \,. \tag{2.5}
$$

These open string quantities are deformation independent, which means that the open string metric and coupling constant are independent of the deformation parameter θ, and that the noncommutativity parameter is a constant, i.e., independent of the radial coordinate r [9, 21, 33]. Together with results in [9] this shows that all types (i.e., electric, magnetic and light-like) of one parameter deformations of Dp-branes give deformation independent open string data. Taking the magnetic near horizon limit gives the following open string data, and Yang-Mills coupling constant:

$$
\begin{aligned}
\frac{G_{\mu\nu}}{\alpha'} &= \frac{1}{\tilde{\ell}^2} \Big(\frac{\tilde{r}}{\tilde{R}} \Big)^{\frac{7-p}{2}} \eta_{\mu\nu} \,, \qquad G^2_{\rm os} = \tilde{g} \Big(\frac{\tilde{R}}{\tilde{r}} \Big)^{\frac{(3-p)(7-p)}{4}} \,, \\
\Theta^{-1} &= \tilde{\ell}^2 \,, \qquad g^2_{\rm YM} = \tilde{g} \tilde{\ell}^{p-3} \,.
\end{aligned} \tag{2.6}
$$

In the solution (2.1), there is a non-zero light-like RR $(d+1) = (p-1)$-form potential, which is dual to the light-like NS-NS B-field. This means that an open Dq-brane description of the physics might be important in certain limits (see section 3.2.5 below). We therefore calculate the open Dq-brane metric and generalized noncommutativity parameter ($(q+1)$-polyvector). For a one-parameter deformation these are given by [33]

$$
\begin{aligned}
G^{{\rm od}q}_{\mu\nu} &= \Big[1 + \frac{1}{(q+1)!} C^2_{q+1} \Big]^{\frac{1-q}{1+q}} \left(g^{{\rm D}q}_{\mu\nu} + \frac{1}{q!} (C^2_{q+1})_{\mu\nu} \right) , \\
\Theta^{\mu_1 \cdots \mu_{q+1}}_{{\rm od}q} &= -(\alpha')^{\frac{q+1}{2}} \Big(1 + \frac{1}{(q+1)!} |C_{q+1}|^2 \Big)^{\frac{q-1}{q+1}} g^{\mu_1\nu_1}_{{\rm D}q} C_{\nu_1 \ldots \nu_{q+1}} G^{\nu_2\mu_2}_{{\rm od}q} \cdots G^{\nu_{q+1}\mu_{q+1}}_{{\rm od}q} \,,
\end{aligned} \tag{2.7}
$$

where $g^{{\rm D}q}_{\mu\nu} = e^{-\frac{2\phi}{q+1}} g_{\mu\nu}$ is the closed Dq-brane metric, and

$$
(C^2_{q+1})_{\mu\nu} = g^{\rho_1\sigma_1}_{{\rm D}q} \cdots g^{\rho_q\sigma_q}_{{\rm D}q} C_{\rho_1 \ldots \rho_q \mu} C_{\sigma_1 \ldots \sigma_q \nu} \,, \qquad C^2_{q+1} = g^{\mu\nu}_{{\rm D}q} (C^2_{q+1})_{\mu\nu} \,. \tag{2.8}
$$

Inserting the above solution (2.1) in (2.7), gives the following *deformation independent* result

$$
G^{{\rm od}q}_{\mu\nu} = (g^2 H)^{-\frac{1}{q+1}} \eta_{\mu\nu} \,, \qquad \Theta^{-3 \ldots (q+2)}_{{\rm od}q} = -g (\alpha')^{\frac{q+1}{2}} \theta \,. \tag{2.9}
$$

[5]Note that the $-$ and 1 in Θ^{-1} are indices and should not be confused with the inverse of Θ, which does not occur at all in this paper.

It is important that we have obtained a deformation independent result, since this gives further credibility to the open D-brane metric and theta parameters obtained in [33]. To be more specific: It gives further evidence that the tensor structure of (2.7) is correct, but unfortunately gives no information on the conformal factor, since $C_{q+1}^2 = 0$ for a light-like deformation. Taking the magnetic near horizon limit gives the following open Dq-brane metric and theta parameter:

$$\frac{G^{\mathrm{od}q}_{\mu\nu}}{\alpha'} = \frac{1}{\tilde{g}^{\frac{2}{q+1}}\tilde{\ell}^2}\Big(\frac{\tilde{r}}{\tilde{R}}\Big)^{\frac{5-q}{q+1}}\eta_{\mu\nu}\,, \qquad \Theta^{-3\ldots(q+2)}_{\mathrm{od}q} = -\tilde{g}\tilde{\ell}^{q+1}\,. \tag{2.10}$$

Note that the open Dq-brane metric diverges in units of α', in the decoupling limit ($\hat{r} \to \infty$), while the generalized noncommutativity parameter is fixed and constant, which is what we expected.

2.2 Deforming an NS5-brane with light-like RR C-field

Next we obtain NS5-brane solutions with a light-like RR $(p+1)$-form turned on. As we will see below, these solutions also contain a light-like RR $(5-p)$-form potential. To obtain these solutions we S-dualize (2.1) for $p = 5$ and then we use T-duality, using the conventions in appendix B. This gives the following result (we get three different solutions labeled by 1,2,3):

$$\begin{aligned}
ds^2 &= 2dx'^- dx'^+ - \theta^2 H^{-1}(dx'^+)^2 + (dx'^1)^2 + (dx'^3)^2 + (dx'^4)^2 + (dx'^5)^2 + \\
&\quad + H(dr'^2 + r'^2 d\Omega_3^2)\,, \quad H = 1 + \frac{N\alpha'}{r'^2}\,, \\
e^{2\phi} &= g'^2 H\,, \\
&\quad 1: p = 0,4 \quad C_+ = -C_{+1345} = g'^{-1}\theta H^{-1}\,, \\
&\quad 2: p = 1,3 \quad C_{+1} = C_{+345} = g'^{-1}\theta H^{-1}\,, \\
&\quad 3: p = 2 \quad C_{+15} = C_{+34} = g'^{-1}\theta H^{-1}\,, \\
\text{where} &\quad g' = g^{-1}\,, \quad x'^\mu = g^{-1/2}x^\mu\,, \quad r' = g^{-1/2} r\,.
\end{aligned} \tag{2.11}$$

These three solutions correspond to the following bound states: 1. (IIA) NS5-D4-D0-W, 2. (IIB) NS5-D3-D1-W and 3. (IIA) NS5-D2-D2-W. Note that 2 is S-dual to D5-D3-F1-W, 1 lifts (plus a rotation) to M5-W (smeared in one direction) and 3 lifts to M5-M2-M2-W (smeared in one direction). Next we take the following modified magnetic near horizon limit (similar to (20) in [30]):

$$\hat{x}^\mu = \frac{\ell}{(\alpha')^{1/2}}x'^\mu\,, \qquad \hat{r} = \frac{\ell^3}{(\alpha')^{3/2}}r'\,, \qquad \hat{g} = g'\frac{\ell^2}{\alpha'}\,, \qquad \ell^2 = \theta\alpha'\,, \quad \alpha' \to 0\,. \tag{2.12}$$

Taking the near horizon limit for the solution (2.11), gives the following supergravity dual:

$$\begin{aligned}
\frac{ds^2}{\alpha'} &= \frac{1}{\ell^2}\Big(2d\hat{x}^- d\hat{x}^+ - (d\hat{x}^+)^2 + \Big(\frac{\hat{r}}{\hat{R}}\Big)^2 (d\hat{x}^1)^2 + (d\hat{x}^3)^2 + \cdots + (d\hat{x}^5)^2 + \\
&\quad + \Big(\frac{\hat{R}}{\hat{r}}\Big)^2 (d\hat{r}^2 + \hat{r}^2 d\Omega_3^2)\Big)\,, \quad \hat{R}^2 = N\ell^2\,,
\end{aligned}$$

$$
\begin{aligned}
e^{2\phi} &= \hat{g}^2\Big(\frac{\hat{R}}{\hat{r}}\Big)^2\,,\\
&1: p=0,4 \quad \frac{C_{+}}{(\alpha')^{1/2}} = \frac{1}{\hat{g}\ell}\Big(\frac{\hat{r}}{\hat{R}}\Big)^2 \quad \frac{C_{+1345}}{(\alpha')^{5/2}} = -\frac{1}{\hat{g}\ell^5}\Big(\frac{\hat{r}}{\hat{R}}\Big)^2\,,\\
&2: p=1,3 \quad \frac{C_{+1}}{\alpha'} = \frac{1}{\hat{g}\ell^2}\Big(\frac{\hat{r}}{\hat{R}}\Big)^2 \quad \frac{C_{+345}}{(\alpha')^{2}} = \frac{1}{\hat{g}\ell^4}\Big(\frac{\hat{r}}{\hat{R}}\Big)^2\,,\\
&3: p=2 \quad \frac{C_{+15}}{(\alpha')^{3/2}} = \frac{C_{+34}}{(\alpha')^{3/2}} = \frac{1}{\hat{g}\ell^3}\Big(\frac{\hat{r}}{\hat{R}}\Big)^2\,.
\end{aligned} \tag{2.13}
$$

These solutions are supergravity duals of Little String Theory with different noncommutative deformations, see section 3.

In this case it is not relevant for our investigations below to calculate open F1-string data since open F1-strings cannot end on NS5-branes, without breaking supersymmetry. However, since open Dp-branes can end on NS5-branes, we can instead calculate the open Dp-brane metric and generalized theta parameter. Inserting the above solution (2.11) in (2.7), gives the following *deformation independent* result[6]

$$
\begin{aligned}
G^{\mathrm{od}p}_{\mu\nu} &= (g'^2 H)^{-\frac{1}{p+1}}\eta_{\mu\nu}\,,\\
&1: p=0,4 \quad \Theta^{-1345}_{\mathrm{od4}} = g'(\alpha')^{5/2}\theta\,,\\
&2: p=1,3 \quad \Theta^{-1}_{\mathrm{od1}} = -g'\alpha'\theta\,, \quad \Theta^{-345}_{\mathrm{od3}} = -g'(\alpha')^2\theta\,,\\
&3: p=2 \quad \Theta^{-15}_{\mathrm{od2}} = \Theta^{-34}_{\mathrm{od2}} = -g'(\alpha')^{3/2}\theta\,.
\end{aligned} \tag{2.14}
$$

Again this is a welcomed result. Taking the magnetic near horizon limit (2.12), gives the following open Dp-brane metric and theta parameter:

$$
\begin{aligned}
\frac{G^{\mathrm{od}p}_{\mu\nu}}{\alpha'} &= \frac{1}{\hat{g}^{\frac{2}{p+1}}\ell^2}\Big(\frac{\hat{r}}{\hat{R}}\Big)^{\frac{2}{p+1}}\eta_{\mu\nu}\,,\\
&1: p=0,4 \quad \Theta^{-1345}_{\mathrm{od4}} = \hat{g}\ell^5\,,\\
&2: p=1,3 \quad \Theta^{-1}_{\mathrm{od1}} = -\hat{g}\ell^2\,, \quad \Theta^{-345}_{\mathrm{od3}} = -\hat{g}\ell^4\,,\\
&3: p=2 \quad \Theta^{-15}_{\mathrm{od2}} = \Theta^{-34}_{\mathrm{od2}} = -\hat{g}\ell^3\,.
\end{aligned} \tag{2.15}
$$

Note that the open Dp-brane metric diverges in units of α', in the decoupling limit ($\hat{r}\to\infty$), while the generalized noncommutativity parameters are fixed and constant, as expected.

2.3 Deforming an M5-brane with light-like three form A

We end this section by deforming an M5-brane with a light-like three form A. This solution is easily obtained by lifting the solution (2.1) for $p=4$, i.e., we lift the bound state D4-D2-F1-W to eleven dimensions, which gives an M5-M2-M2-W bound state, where the M2-branes have the same charge. Lifting the solution (2.1) for $p=4$, to eleven dimensions

[6] In case 3 we have to use the open membrane data (2.19) below (since we have the special case with a self-dual three form), but of course use the closed D2-brane metric instead of the closed membrane metric, see [33]. Also let $\ell_{\mathrm{p}}^3 \to (\alpha')^{3/2}$.

gives the following bound state (we label the uplifting direction by x^5):

$$
\begin{aligned}
ds^2 &= H^{-1/3}(2dx^-dx^+ - \theta^2 H^{-1}(dx^+)^2 + (dx^1)^2 + (dx^3)^2 + (dx^4)^2 + (dx^5)^2) + \\
&\quad + H^{2/3}(dr^2 + r^2 d\Omega_4^2)\,, \\
A_{+15} &= -\theta H^{-1}\,, \quad A_{+34} = \theta H^{-1}\,, \quad H = 1 + \frac{N\ell_{\rm p}^3}{r^3}\,, \qquad (2.16)
\end{aligned}
$$

where we have used (A.2) in appendix A. Next, by 'lifting' the near horizon limit (2.2) we obtain the following eleven dimensional near horizon limit, where

$$
x^\mu\,, \qquad \tilde{r} = \frac{\ell_{\rm m}^3}{\ell_{\rm p}^3} r\,, \qquad \ell_{\rm m}^3 = \theta \ell_{\rm p}^3\,, \qquad (2.17)
$$

are kept fixed in the $\ell_{\rm p} \to 0$ limit. Note that this near horizon limit is a 'tensor theory type' of limit [42], since we keep $r/\ell_{\rm p}^3$ fixed. We will also see below that this limit gives the supergravity dual of the (2,0) tensor theory with a light-like deformation. Taking this near horizon limit gives the following solution

$$
\begin{aligned}
\frac{ds^2}{\ell_{\rm p}^2} &= \frac{1}{\ell_{\rm m}^2}\Big(\frac{\tilde{r}}{\tilde{R}}\Big)\Big[2dx^-dx^+ - \Big(\frac{\tilde{r}}{\tilde{R}}\Big)^3 (dx^+)^2 + (dx^1)^2 + (dx^3)^2 + (dx^4)^2 + (dx^5)^2\Big] + \\
&\quad + \frac{1}{\ell_{\rm m}^2}\Big(\frac{\tilde{R}}{\tilde{r}}\Big)^2 (d\tilde{r}^2 + \tilde{r}^2 d\Omega_4^2)\,, \\
\frac{A_{+15}}{\ell_{\rm p}^3} &= -\frac{A_{+34}}{\ell_{\rm p}^3} = -\frac{1}{\ell_{\rm m}^3}\Big(\frac{\tilde{r}}{\tilde{R}}\Big)^3\,, \qquad \tilde{R}^3 = N\ell_{\rm m}^3\,. \qquad (2.18)
\end{aligned}
$$

We continue by calculating the open membrane metric and generalized noncommutativity parameter, which for a general background are given by [32, 33, 38]:

$$
\begin{aligned}
G_{\mu\nu}^{\rm OM} &= \Big(\frac{1-\sqrt{1-K^{-2}}}{K^2}\Big)^{1/3}\Big(g_{\mu\nu} + \frac{1}{4}A_{\mu\nu}^2\Big)\,, \\
\Theta_{\rm OM}^{\mu\nu\rho} &= -\ell_{\rm p}^3[K(1-\sqrt{1-K^{-2}})]^{2/3} g^{\mu\mu_1} A_{\mu_1\nu_1\rho_1} G_{\rm OM}^{\nu_1\nu} G_{\rm OM}^{\rho_1\rho}\,, \\
K &= \sqrt{1+\frac{1}{24}A^2}\,, \quad A_{\mu\nu}^2 = g^{\mu_1\nu_1} g^{\mu_2\nu_2} A_{\mu_1\mu_2\mu} A_{\nu_1\nu_2\nu}\,, \quad A^2 = g^{\mu\nu} A_{\mu\nu}^2\,. \qquad (2.19)
\end{aligned}
$$

Calculating the open membrane metric and generalized noncommutativity parameter for (2.16), gives the following *deformation independent* result:

$$
G_{\mu\nu}^{\rm OM} = H^{-1/3}\eta_{\mu\nu}\,, \qquad \Theta_{\rm OM}^{-15} = -\Theta_{\rm OM}^{-34} = \ell_{\rm p}^3\theta\,. \qquad (2.20)
$$

By taking the magnetic near horizon limit (2.17), we obtain the following open membrane metric and generalized noncommutativity parameter:

$$
\frac{G_{\mu\nu}^{\rm OM}}{\ell_{\rm p}^2} = \frac{1}{\ell_{\rm m}^2}\Big(\frac{\tilde{r}}{\tilde{R}}\Big)\eta_{\mu\nu}\,, \qquad \Theta_{\rm OM}^{-15} = -\Theta_{\rm OM}^{-34} = \ell_{\rm m}^3\,, \qquad (2.21)
$$

which implies that we have a fixed generalized noncommutativity parameter and that the open membrane metric diverges in unites of $\ell_{\rm p}^2$, in the decoupling limit ($\tilde{r} \to \infty$).

We have seen in this section that for light-like deformations of Dp/NS5/M5-branes, we obtain deformation independent open brane (F1/Dq/M2) data in *all* cases, before we take any near horizon limit. For the open Dq-brane and open membrane data this is good news, since it gives further evidence that the tensor structure of the expressions for the open brane metrics and generalized noncommutativity parameters, obtained in [32, 33, 38], are correct. Further indications are also given by the fact that we in *all* cases obtain open brane metrics which diverges in units of α' (or $\ell_{\rm p}^2$), when $\tilde{r} \to \infty$ (decoupling limit), after taking the near horizon limit. For the open string case this implies that massive open strings decouple and that we obtain a field theory on the probe brane. Similar for the open D/membrane cases we expect it to imply that there are no light open D/membranes in the world volume theories.

Together with the results in [33] (electric and magnetic cases), the results in this section confirm that *all* types of one parameter deformations (i.e., electric, magnetic and light-like), in all cases (i.e., deformations of Dp/NS5/M5-branes) give deformation independent open brane metrics and fixed noncommutativity parameters.

3. Light-like noncommutativity and a duality web

In this section we will discuss the various noncommutative theories, for which we obtained supergravity duals and relevant open F1/Dq/M2 data in the last section (see also [11, 12]). In particular, we will be interested in the different phases of the theories and their duality relations (for IIB branes S-duality and lift to M-theory for IIA branes). We will also, in subsection 3.3, include a few comments on the relation between open membrane data in six dimensions and open string data in five dimensions.

3.1 The light-like (2,0) tensor theory

The supergravity dual (2.18) corresponds to an M5-brane with a light-like three form A turned on. The world volume theory is conjectured to be [34, 35, 36] the six dimensional (2,0) tensor theory with some kind of generalized noncommutativity structure, characterized by a three form $\Theta_{\rm OM}$ [35]. This 'light-like' (2,0) tensor theory (LL(2,0)) is obtained from the (2,0) theory by perturbing the (2,0) theory with a dimension 9 (mass^9) operator [34, 35, 36]. For a further discussion about the DLCQ description of this theory see [34, 35, 36].

In the last section we computed the open membrane metric and generalized noncommutativity parameter (2.21), for the supergravity dual (2.18). We see that we have obtained a fixed generalized noncommutativity parameter, which confirms the existence of the conjectured three form in [35]. However, it is not yet clear what this generalized noncommutativity parameter implies for the physics of the deformed M5-brane. It has been speculated [37, 33, 38], that it gives rise to some kind of noncommutative loop-space structure and/or non-associative geometry. This three form might also be connected to a three point function, similar to the open string noncommutative parameter which is connected to the open string two-point function [3]. Also, from the three form generalized noncommutativity parameter we see that there is a length scale $\ell_{\rm m}$ in the theory where the 'noncommutative' effects become important. This implies that for energies $E \ll 1/\ell_{\rm m}$, the (2,0) theory is a good description of the physics, while for energies $E \sim 1/\ell_{\rm m}$ and above, we have to use the LL(2,0) theory.

Equation (2.21) implies that the open membrane metric diverges in units of $\ell_{\rm p}^2$, in the decoupling limit. This also happens if we calculate the open membrane metric for the supergravity dual of the 'ordinary' (2,0) theory. However, it is very different from the OM-theory [23, 24] case. For the OM-theory supergravity dual (see e.g. [33]), the open membrane metric is instead fixed in unites of $\ell_{\rm p}^2$. This result, and the fact that the LL(2,0) theory can be obtained as a perturbation of the (2,0) theory, indicates that the 'ordinary' (2,0) tensor theory and the light-like version are rather 'similar' in the sense that they have a similar amount of degrees of freedom (d.o.f.). OM-theory instead, is expected to be very different compared to these tensor theories, with a much larger amount of d.o.f, see also discussions in [11, 36]. Because of this 'similarity' of the two tensor theories, it would be very interesting to investigate if it is possible to obtain a map, similar to the Seiberg-Witten map [3], for the LL(2,0) theory. A map of this kind might involve some kind of generalization of the star product [3], where the three form $\Theta_{\rm OM}$ is important analogous to how the open string noncommutativity parameter Θ is important in the Seiberg-Witten map.

In the next subsection 3.1 we will show how the light-like three form $\Theta_{\rm OM}$ is related to the five dimensional NCYM light-like noncommutativity parameter Θ.

3.2 Light-like theories and duality relations

In this subsection, we continue by analyzing various non-gravitational theories with light-like noncommutativity in three to six dimensions, using the supergravity duals and open brane data, that we obtained for the deformed Dp/NS5/M5-branes in the last section. We begin in three dimensions.

3.2.1 Light-like noncommutativity in three dimensions

The world volume theory of a D2-brane (probe brane) with a light-like B-field, in the decoupling limit, is a U(1) NCYM theory with $\Theta^{-1} = \tilde{\ell}^2$. The supergravity dual is given in (2.3) with $p = 2$. The divergence of the open string metric (in units of α') implies that massive open string modes decouple. This theory has a dimension-full Yang-Mills coupling constant $g_{\rm YM}^2 = \tilde{g}\tilde{\ell}^{-1}$, which means, as usual, that the theory has to be completed at low energies $E \leq g_{\rm YM}^2$. At low energies, we expect the light-like NCYM theory to be completed by an SO(8) invariant super conformal M2-brane theory (the bound state D2-F1-D0-W lifts (plus a rotation) to M2-W, where the wave is expected to be irrelevant at low energies).

3.2.2 Light-like noncommutativity in four dimensions

Next, in four dimensions (see also [11]–[14]), we have a D3-brane (probe brane) with a light-like B-field (dual to a light-like C_2-field). The supergravity dual is given by (2.1), with $p = 3$, and the open string data is given in (2.6). The open string metric diverges, in unites of α', in the decoupling limit, while the noncommutativity parameter is fixed $\Theta^{-1} = \tilde{\ell}^2$. This implies that we have obtained a U(1) noncommutative Yang-Mills theory, with light-like noncommutativity and dimensionless Yang-Mills coupling constant $g^2_{\rm YM} = \tilde{g}$. This theory is unitary and renormalizable, which has been shown in [11]. It was also argued that the strong coupling limit of this theory is another NCYM theory with light-like noncommutativity (see also [12, 14]). The relations between the two theories coupling constants and noncommutativity parameters are given by

$$g^2_{\rm YM(s)} = \frac{1}{g^2_{\rm YM}}\,, \quad \Theta^{-3}_{\rm (s)} = -g^2_{\rm YM}\Theta^{-1}\,, \tag{3.1}$$

where the index (s) means the S-dual parameter. To obtain this result, we S-dualize the supergravity dual, calculate the open string data, and compare the open string data for the two S-dual supergravity duals. Note that this is rather different compared to NCYM with space-space noncommutativity, which is S-dual to NCOS in four dimensions [15].

The above NCYM theory with non-zero Θ^{-1}, can be generalized to light-like NCYM with both Θ^{-1} and Θ^{-3} non-zero. The S-dual of this theory is given by a light-like NCYM theory which also has non-zero $\Theta^{-1}_{\rm (s)}$ and $\Theta^{-3}_{\rm (s)}$. If $\Theta^{-1} = \Theta^{-3} = \tilde{\ell}^2$, the parameters of the two (S-dual) theories are related as follows: $g^2_{\rm YM(s)} = \frac{1}{g^2_{\rm YM}}$ and $\Theta^{-1}_{\rm (s)} = -\Theta^{-3}_{\rm (s)} = g^2_{\rm YM}\tilde{\ell}^2$.

3.2.3 Light-like noncommutativity in five dimensions

In five dimensions we have a D4-brane with a light-like B-field (dual to a light-like C_3-field). In this case we also have a U(1) NCYM theory with $\Theta^{-1} = \tilde{\ell}^2$, but with a dimension-full coupling constant $g^2_{\rm YM} = \tilde{g}\tilde{\ell}$. This implies that the theory is not renormalizable, i.e., the theory breaks down above energies $E \sim (g^2_{\rm YM})^{-1}$. This can be seen if we introduce an effective dimensionless coupling constant $g^2 = Eg^2_{\rm YM}$, where E is the energy scale. The effective coupling constant g^2 is only much less then one for energies $E \ll 1/g^2_{\rm YM}$. For larger energies this theory can be completed by the world volume theory of an M5-brane with a light-like three form A-field turned on, wrapped on a circle parallel to the M5-brane. This world volume theory is the light-like (2,0) theory with generalized noncommutativity parameter $\Theta^{-15}_{\rm OM} = -\Theta^{-34}_{\rm OM}$, see section 3.1. Wrapping the M5-brane on a circle with radius R (in rescaled units), gives the following relations between the parameters of the two theories

$$g^2_{\rm YM} = R\,, \qquad \Theta^{-1} = \frac{\Theta^{-15}_{\rm OM}}{R}\,. \tag{3.2}$$

These relations are obtained by comparing the two supergravity duals (2.3) with $p = 4$, and (2.18), using the results in appendix A. Equation (3.2) implies that we have the following two 'phase diagrams'[7] for a D4-brane with a light-like B-field turned on, after taking the decoupling limit (see also subsection 3.3 for additional comments on this case):

1. $\Theta^{1/3}_{\rm OM} \ll \Theta^{1/2}$: phase 1. for energies $E \ll 1/\Theta^{1/2}$ the physics on the probe brane can be described by 'ordinary' 5 dimensional super Yang-Mills, since the noncommutativity

[7] What we mean here by phase diagram is that we start at low energy and let the energy E become larger and larger and at the same time we give the description (theory) which is relevant at a certain energy. At certain energies the description of the physics has to be changed, since new effects appear (e.g. noncommutativity). Note that strictly speaking a transition from one description to another does not have to be a phase transition. For example, going from SYM to NCYM is not a 'true' phase transition.

is negligible; phase 2. for energies above $1/\Theta^{1/2}$ but much less then $1/g_{\rm YM}^2 = 1/R$, light-like NCYM provides a good description; phase 3. for energies $E \sim 1/g_{\rm YM}^2 = 1/R$ and above, we have to use the light-like (2,0) tensor theory on a circle with radius R.

2. $\Theta_{\rm OM}^{1/3} \gg \Theta^{1/2}$: Start with 'ordinary' SYM for energies $E \ll 1/R$, while for energies $E \sim 1/R$ we have to use the (2,0) tensor theory on circle. For energies $E \sim 1/\Theta_{\rm OM}^{1/3}$ and above we use the light-like (2,0) tensor theory on a circle. Note that there is no NCYM phase in this case.

3.2.4 Light-like noncommutativity in six dimensions

Type IIB D5/NS5-branes. In six dimensions, the world volume theory for a D5-brane with light-like B-field (dual to a RR C_4-field), is NCYM theory with noncommutativity parameter $\Theta^{-1} = \tilde{\ell}^2$ and dimension-full coupling constant $g_{\rm YM}^2 = \tilde{g}\tilde{\ell}^2$ (the supergravity dual is given in (2.3) with $p = 5$). This theory is not renormalizable, and has to be completed for energies above $E \sim 1/g_{\rm YM}$, which is seen from the effective coupling constant $g^2 = g_{\rm YM}^2 E^2$. The six dimensional light-like NCYM theory can be completed by (1,1) Little String theory with light-like noncommutativity (light-like LST). To obtain the relevant phase diagram we use the supergravity duals (2.3) with $p = 5$, and (2.13) case 2, which are S-dual to each other[8]. We have to use the S-dual 'picture' for energies above $E \sim 1/g_{\rm YM}$, since light-like NCYM becomes strongly coupled. The relations between the parameters of the two supergravity duals, as well as the relations between the relevant open brane quantities are given by

$$\hat{g} = \frac{1}{\tilde{g}}\,, \qquad \ell^2 = \tilde{g}\tilde{\ell}^2 \qquad \Theta_{\rm od1}^{-1} = -\Theta^{-1}\,. \tag{3.3}$$

The tension of the little strings on the NS5-brane is given by

$$T_{\rm s} = \frac{1}{g_{\rm YM}^2} = \frac{1}{\tilde{g}\tilde{\ell}^2} = \frac{1}{\ell^2}\,, \tag{3.4}$$

where we have used (3.3) and $g_{\rm YM}^2$ is the six dimensional Yang-Mills coupling constant. Note that we in this paper ignore factors of 2π in the definition of tension. Before we obtain the phase diagrams, we note that for the deformed NS5-brane we have both a two form RR field as well as a four form RR field, in the supergravity dual. These RR fields are not independent but are dual to each other. This duality is due to the self-duality of the M-theory three form on the M5-brane. The RR two form leads to a noncommutativity parameter $\Theta_{\rm od1}$, while the four form leads to a generalized noncommutativity parameter $\Theta_{\rm od3}$, see (2.15) case 2. Note also that the length scales where these noncommutativity parameters become important are different. This implies that they are important at different energy scales. However, we expect both to be important at very high energies (i.e., energies E larger then both $1/\Theta_{\rm od1}^{1/2}$ and $1/\Theta_{\rm od3}^{1/4}$). In [45, 11] a possible DLCQ description

[8] The S-dual of the IIB D5-brane is the IIB NS5-brane. For an NS5-brane without any RR-fields turned on, the physics (after decoupling gravity) at energies $E \sim T_{\rm s}^{1/2}$ and above, is described by the 'ordinary' Little String theory [43, 44], i.e., (1,1) LST theory for the IIB NS5-brane and (2,0) LST theory for the IIA NS5-brane. For energies $E \ll T_{\rm s}^{1/2}$ the effective theory is (1,1) six dimensional SYM for IIB and a (2,0) tensor theory for IIA. For a DLCQ description of these theories, see [45].

of this light-like LST were discussed. However only the RR two form were considered. The exact role of the RR four form and $\Theta_{\rm od3}$ are unclear to us at the moment, but we expect them to play some part in a complete description of the above light-like LST. Using (3.3) and (3.4), we obtain the following two phase diagrams:

1. $\Theta^{1/2} \ll g_{\rm YM}$. If we start at low energy $E \ll 1/g_{\rm YM}$, the D5-brane (probe brane) physics can be described by 'ordinary' six dimensional SYM. At energies $E \sim 1/g_{\rm YM}$ $= T_{\rm s}^{1/2}$ we have to use the S-dual description (D5$\rightarrow$ NS5). This means that we can use 'ordinary' (1,1) Little String theory [43] as long as $E \ll 1/(\Theta_{\rm od3})^{1/4}$. Above this energy we have to use light-like (1,1) LST theory where $\Theta_{\rm od3}^{-345} = \hat{g}\ell^4$ is important. For energies $E \sim 1/(\Theta_{\rm od1})^{1/2}$ and above also the noncommutativity parameter $\Theta_{\rm od1}^{-1} = \hat{g}\ell^2$ becomes important.

2. $\Theta^{1/2} \gg g_{\rm YM}$. We start at low energy $E \ll 1/(\Theta)^{1/2}$, where we have 'ordinary' six dimensional SYM. For energies above this but much less then $1/g_{\rm YM}$ we have light-like NCYM. At energies $E \sim 1/g_{\rm YM}$ we switch to the S-dual description, which implies that we have light-like (1,1) LST theory where both $\Theta_{\rm od3}$ and $\Theta_{\rm od1}$ are relevant.

Type IIA NS5-brane. Next, we consider the type IIA NS5-brane with light-like RR three form turned on (the supergravity dual is given by (2.13) case 3). A light-like three form means that we have (2,0) LST theory with generalized noncommutativity parameter $\Theta_{\rm od2}^{-15} = \Theta_{\rm od2}^{-34} = -\hat{g}\ell^3$.

The type IIA NS5-brane with light-like RR three form, can be interpreted as an M-theory M5-brane with light-like three form turned on, with a small transverse circle with radius $R_{\rm T}$ (in rescaled units). The relations between the relevant open D2/M2 data, are given by

$$\ell_{\rm m}^3 = \hat{g}\ell^3\,, \qquad \Theta_{\rm od2} = \Theta_{\rm OM}\,, \qquad T_{\rm s} = \frac{R_{\rm T}}{\ell_{\rm m}^3} = \frac{1}{\ell^2}\,. \tag{3.5}$$

These relations implies that we have the following two phase diagrams for the type IIA NS5-brane with light-like three form, after taking the decoupling limit:

1. $R_{\rm T} \ll \ell_{\rm m} \ll \ell$. At low energy $E \ll T_{\rm s}^{1/2}$ we have the (2,0) NS5 tensor theory as usual. For energies $E \sim T_{\rm s}^{1/2}$ but much less then $1/\Theta_{\rm od2}^{1/3}$, the physics on the probe brane is described by the 'ordinary' LST, while for energies $E \sim 1/\Theta_{\rm od2}^{1/3}$ and above we have to use the light-like LST with $\Theta_{\rm od2}^{-15} = \Theta_{\rm od2}^{-34}$.

2. $R_{\rm T} \gg \ell_{\rm m} \gg \ell$. Start with the (2,0) M5 tensor theory with a transverse circle with radius R, for energies $E \ll 1/\Theta_{\rm OM}^{1/3} = 1/\ell_{\rm m}$. Then for energies above this but much less then $T_{\rm s}^{1/2}$, we use the light-like (2,0) tensor theory, while for energies $E \sim T_{\rm s}^{1/2}$ and above, we use the light-like LST with $\Theta_{\rm od2}^{-15} = \Theta_{\rm od2}^{-34}$.

For a discussion about a possible DLCQ description of this light-like LST, see [45, 11].

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For the IIA NS5-brane with light-like RR five form (dual to a RR one form) turned on we have light-like LST with generalized noncommutativity parameter $\Theta_{\rm od4}^{-1345} = \hat{g}\ell^4$ (the supergravity dual is given by (2.13) case 1). We do not expect the dual one form to lead to any noncommutativity. The two phase diagrams are as follows:

1. $\Theta_{\rm od4}^{1/5} \ll T_{\rm s}^{-1/2}$. We start at low energies $E \ll T_{\rm s}^{1/2}$, where we have the 'ordinary' (2,0) NS5 tensor theory. At $E \sim T_{\rm s}^{1/2}$ we use 'ordinary' LST, while for energies $E \sim 1/\Theta_{\rm od4}^{1/5}$ and above, we have to use the light-like LST with generalized noncommutativity parameter $\Theta_{\rm od4}^{-1345}$.

2. $\Theta_{\rm od4}^{1/5} \gg T_{\rm s}^{-1/2}$. At low energies $E \ll T^{1/2}$ we have the (2,0) M5 tensor theory with a transverse circle with radius $R = \hat{g}\ell$, while at energies $E \sim T^{1/2}$ and above, we have the light-like LST with $\Theta_{\rm od4}^{-1345}$.

3.2.5 Light-like noncommutativity and generalized gauge theories

Before we end this subsection, we would like to make a few comments on a possible dual description of the Dp-branes with light-like B-field. As we have seen before, using open strings to define the Dp-branes led to NCYM theories with light-like noncommutativity. On a Dp-brane with light-like B-field there is also a dual light-like $(p-1)$-form RR C-field, as can be seen from the supergravity duals (2.3). This implies that there, in principle, is a dual formulation of the Dp-brane with open Dq-branes ($q = p - 2$) ending on the Dp-brane. From (2.10) we see that the open Dq-brane metric diverges, in units of α', in the decoupling limit, and that there is a fixed generalized noncommutativity parameter $\Theta_{{\rm od}q}^{-3\ldots(q+2)} \neq 0$. This indicates that we can describe the physics on the probe Dp-brane with some kind of generalized gauge theory with generalized noncommutativity parameter $\Theta_{{\rm od}q}$. For $q = 1$ we have 'ordinary' noncommutativity, while for $q > 1$, there is some generalization of noncommutativity. We expect that these theories, if they indeed exists, can be formulated in a similar way as the generalized gauge theories Dq-GT, defined in [30] (which are obtained by deforming a Dp-brane with a magnetic RR $(p-1)$ form). Defining them in a similar way as in [30], gives a gauge coupling which is the same as the Yang-Mills coupling $g_{\rm YM}^2$ and a fixed generalized noncommutativity parameter $\Theta_{{\rm od}q}$, see (2.10). This implies that these generalized gauge theories, similar to NCYM, only are complete theories for $p = 3$ ($q = 1$).

It is unclear to us if these generalized gauge theories are important or not. For example, they can not be used to 'complete' NCYM for $p \neq 3$. Studying them further might, however, give important information on the generalized noncommutative structure they seem to inhibit. Since the relevance of these theories are not clear to us and the fact that we have a well understood NCYM description in these cases, we have chosen to not include them in the phase diagrams for the deformed Dp-branes above.

3.3 Relations between open string/membrane data

In this subsection we will make a few comments on the relations between open string data in five dimensions and open membrane data in six dimensions. As we mentioned before, in [32, 33, 38] the complete open membrane metric was obtained, as well as an open membrane generalized noncommutativity parameter [33, 38]. Further, in [33, 38] it was shown that if one tries to relate the open string metric and coupling constant in five dimensions, to the open membrane metric in six dimensions, there are a few subtleties. In particular, if one reduces the open membrane metric on an 'electric' circle, then the open membrane metric reduces to the open string metric and coupling constant (see [33, 38] for details). With electric reduction we mean the following: Assume the M-theory three form A has nonzero A_{012} and A_{345} components if we use an $\mathrm{SO}(1,5)/\,\mathrm{SO}(1,2) \times \mathrm{SO}(3)$ parametrization (see [33] for details) then an electric reduction implies that the reduction is in the x^2 direction, i.e., $A_{012} \to B_{01}$. Similarly, a reduction in e.g. the x^5 direction is called a magnetic reduction. If one instead reduce on a 'magnetic circle', then it does not seem possible to obtain the open string metric and coupling constant. However, in [33] it was shown that the open membrane metric reduces to an open D2-brane metric (also obtained in [33] with another method as well) in the magnetic case. These results are in fact natural considering that in the electric case the electric component of three form reduces to an electric component of the NS-NS B-field, while in the magnetic case it reduces to an electric component of the RR three form. For a general rank four reduction, it was shown in [38] that it is not possible to obtain the open string metric and coupling constant from the open membrane metric, which also seems natural since the rank 4 reduction is a mixture of an 'electric' *and* a 'magnetic' reduction. For a rank 4 reduction it also does not seem to be possible to obtain an open D2-brane metric. It would be very interesting to further study the rank 4 case, since it is unclear what the failure of the reduction implies for what kind of description one should use on the D4-brane, in this case. It is possible that both open strings and open D2-branes are important for the description of the world volume physics on the D4-brane in this case.

In this paper, we are in particular interested in how the open string data are related to the open membrane data if the M-theory three form is light-like on the M5-brane and reduces to a light-like two form B on the D4-brane (rank 2). This case was not investigated in [33, 38]. For the light-like case we know that A^2, which is defined in (2.19) is $A^2 = 0$ (implies $B^2 = B^4 = 0$). Next, if we use the reduction formulas derived in [33], we obtain that the open membrane metric and generalized noncommutativity parameter *can* be related to the open string metric, coupling constant and noncommutativity parameter, as follows:

$$\frac{G^{\mathrm{OM}}_{\mu\nu}}{\ell_{\mathrm{p}}^2} = (G_{\mathrm{os}}^2)^{-2/3}\frac{G_{\mu\nu}}{\alpha'}\,, \qquad \frac{G^{\mathrm{OM}}_{yy}}{\ell_{\mathrm{p}}^2} = \frac{(G_{\mathrm{os}}^2)^{4/3}}{R^2}\,, \qquad \frac{\Theta^{\mu\nu y}_{\mathrm{OM}}}{R} = \Theta^{\mu\nu}\,, \tag{3.6}$$

where μ, ν are the five dimensional indices and y is the direction in which we reduce. As expected the open brane quantities (3.6) are related analogously to how the closed brane quantities (see (A.2) below) are related. This implies that we can reduce the open membrane data to open string data, in the light-like case. The open D2-brane metric can also be obtained with a similar argument. The crucial fact in that these reductions work, is that $A^2 = 0$, because this implies that the conformal factor in the open membrane metric (2.19) is equal to 1. The results are also further confirmed by comparing the open string/membrane data in (2.5) $p=4$, and (2.20), using $\ell_{\rm p}^2 = g^{2/3}\alpha'$ and $R^2 = g^2\alpha'$.

4. NCYM with space-space noncommutativity and strong coupling

In this section,[9] we will obtain the theory which is the strong coupling (high energy) limit of five dimensional NCYM with $\Theta^{12} = \Theta^{34} \neq 0$. We begin by showing that the light-like (2,0) theory is not the strong coupling (high energy) limit of NCYM with $\Theta^{12} = \Theta^{34} \neq 0$. Naively one might think that the two theories are connected, since the bound state D4-D2-D2-D0, which is used to obtain the NCYM supergravity dual, lifts to an M5-M2-M2-W bound state. However, as we will see below, the supergravity solutions are in fact *not* related, once we take the near horizon limit. The relevant part of the supergravity solution, corresponding to the bound state D4-D2-D2-D0 (with equal D2-brane charge), is given by[10]

$$\begin{aligned} ds^2 &= H^{-1/2}\Big(-(dx^0)^2 + \frac{1}{h}((dx^1)^2 + \cdots + (dx^4)^2)\Big) + H^{1/2}(dr^2 + r^2 d\Omega_4^2)\,, \\ e^{2\phi} &= g^2 H^{-1/2} h^{-2}\,, \qquad B_{12} = B_{34} = gC_{012} = gC_{034} = -\theta'(Hh)^{-1}\,, \\ gC_0 &= \theta'^2 H^{-1}\,, \qquad h = 1 + \theta'^2 H^{-1}\,. \end{aligned} \tag{4.1}$$

If we continue by taking the near horizon limit (2.2) (with θ' instead of θ), we obtain the supergravity dual of five dimensional NCYM with Yang-Mills coupling constant and noncommutativity parameter given by

$$g_{\rm YM}^2 = \tilde{g}\tilde{\ell}\,, \qquad \Theta^{12} = \Theta^{34} = \tilde{\ell}^2\,. \tag{4.2}$$

For energies above $E \sim 1/g_{\rm YM}^2$ this NCYM theory is strongly coupled and we have to change description. We therefore lift the type IIA solution to M-theory, in order to find a theory which 'completes' the above NCYM theory. Lifting the type IIA D4-D2-D2-D0 bound state to M-theory gives the following M5-M2-M2-MW bound state[11]

$$\begin{aligned} ds^2 &= (Hh)^{-1/3}[2dx^- dx^+ - 2\theta'^2 (Hh)^{-1}(dx^+)^2 + (dx^1)^2 + \cdots + (dx^4)^2] + \\ &\quad + (Hh)^{2/3}(dr^2 + r^2 d\Omega_4^2)\,, \quad x^{\pm} = \frac{1}{\sqrt{2}}(x^5 \pm x^0)\,, \\ A_{+12} &= A_{+34} = -\sqrt{2}\theta'(Hh)^{-1}\,, \quad h = 1 + \theta'^2 H^{-1}\,, \qquad H = 1 + \frac{N\ell_{\rm p}^3}{r^3}\,. \end{aligned} \tag{4.3}$$

Comparing this solution with (2.16) in section 2, we see that (4.3) also correspond to an M5-M2-M2-MW bound state, since $H' = Hh = 1 + \theta'^2 + \frac{N\ell_{\rm p}^3}{r^3}$ is a harmonic function. Note that the two solutions are very similar, except that the constants in the harmonic function

[9] To obtain the results in this section we have benefited from discussions with M. Cederwall, U. Gran, B.E.W. Nilsson and P. Sundell.

[10] We use the $O(p+1,p+1)$ method in [10, 9, 21]. This bound state have been obtained earlier in [5], using other methods. Note that we have not included the RR five form.

[11] We use the relations in appendix A, as well as rewriting the solution in light-like coordinates. This bound state solution has earlier been obtained, using different methods, in [12, 46, 47, 48].

are different. This difference in the harmonic functions is, however, very important when we take the near horizon limit (2.17), because the constant in the harmonic function H' has a θ' dependence, where θ' scales (2.17). This implies that after we have taken the near horizon limit (2.17) in (2.16) and (4.3), we obtain two *different* results. This naturally also leads to different results for the open membrane metric. Looking closer, we find that in the form the solution (4.3) is written, the limit (2.17) is 'trivial'. To be more precise: taking the limit (2.17) for the solution (4.3), gives a solution which still is a *complete* M5-M2-M2-MW solution (let $\ell_{\rm m} \to \ell_{\rm p}$, which gives (4.3) with $\theta' = 1$, and $H' = Hh = 1 + \frac{N\ell_{\rm p}^3}{r^3}$). The conclusion of this is that the supergravity dual of five dimensional NCYM with rank 4 space-space noncommutativity and the supergravity dual (2.18) of the light-like (2,0) theory are not connected. This can also be seen by reducing the supergravity dual (2.18), in a direction x^2 ($x^2 = \frac{1}{\sqrt{2}}(x^+ + x^-)$), which gives a type IIA supergravity solution which has a singularity in the metric at finite $\tilde{r}$. This solution is clearly not the same as (4.1) after one has taken the near horizon limit (2.2). For related results concerning the M5-M2-M2-MW bound state and near horizon limits, see [48].

We still have not answered the question about the strong coupling limit of five dimensional NCYM with $\Theta^{12} = \Theta^{34} \neq 0$. It is clear from the above discussion, that it can not be the six dimensional light-like (2,0) theory.

Next, the ten dimensional supergravity dual lifts to an eleven dimensional supergravity solution, which approaches flat eleven dimensional space-time, in the $\tilde{r} \to \infty$ limit. In the $\tilde{r} \to \infty$ limit we get the following metric (after rescaling the coordinates)

$$ds^2 = -(dx^0)^2 - dx^0 dx^5 + (dx_9)^2 \,, \tag{4.4}$$

where dx_9 is the line element of nine dimensional flat space and x^5 is the 'uplifted' direction such that $x^5 \sim x^5 + 2\pi R$. The result we get here is similar to what happens if one lifts the NS5-D0 bound state followed by taking the near horizon limit, see [23]. The NS5-D0 bound state is used to obtain the supergravity dual of the OD0 theory [23, 12] (a six dimensional theory containing light D0-branes), which can be obtained as a DLCQ compactification of M-theory with n units of DLCQ momentum in the presence of a transverse M5-brane. If we compare this to our case we see that there are some similarities. For example, in the D4-D2-D2-D0 bound state there is a D0-brane. As in the NS5-D0 case this D0-brane is light in the decoupling limit, with tension (mass) given by $T_{\rm D0} = \frac{1}{\tilde{g}\tilde{\ell}}$. Note that $T_{\rm D0} = 1/g_{\rm YM}^2$. We expect that these light D0-branes can be found as noncommutative solitons (see e.g. [49] for an introduction to noncommutative solitons) in the five dimensional NCYM theory. It would be interesting to explicitly obtain this noncommutative soliton solution in the NCYM theory.

With the above results and comparing with the NS5-D0 case [23], we make the following observation. Since the five dimensional NCYM theory with $\Theta^{12} = \Theta^{34} = \Theta$ is only a low energy effective theory we propose that the world volume theory of the above deformed D4-brane at high energies (i.e., energies $E \sim 1/g_{\rm YM}^2$ and above) is given by a DLCQ compactification of M-theory in the presence of a 'parallel' M5-brane (i.e., one of the M5-brane directions is the compactified x^5 direction) and two transverse M2-branes. The relations between the NCYM data ($g^2_{\rm YM}$ and Θ), the compactification radius R and the effective M-theory Planck mass (in rescaled units) are given by:

$$R = g^2_{\rm YM}\,, \qquad M^3_{\rm eff} = \frac{1}{g^2_{\rm YM}\Theta}\,. \tag{4.5}$$

This gives the following two phase diagrams:

1. $g^2_{\rm YM} \ll \Theta^{1/2}$. Start with 'ordinary' super-Yang-Mills theory up to energies $E \ll 1/\Theta^{1/2}$, followed by NCYM with $\Theta^{12} = \Theta^{34} = \Theta$ for energies $E \ll 1/R = 1/g^2_{\rm YM}$. At energies $E \sim 1/R$ we have to lift to M-theory (i.e., use the above DLCQ compactification of M-theory), since the NCYM theory is strongly coupled.

2. $g^2_{\rm YM} \gg \Theta^{1/2}$. This case is the same as case 1 except that there is *no* NCYM phase.

Note that since the NCYM theory is well defined as long as $E \ll 1/R = 1/g^2_{\rm YM} = T_{\rm D0}$ it is not necessary to use any type IIA description based on light D0-branes in any phase.

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A. Conventions for S-duality, T-duality and lift to eleven dimensions

In this appendix we give the conventions we have used for S-duality, T-duality and lift of type IIA solutions to eleven dimensions.

The S-duality rules for a type IIB solution (in the string frame) are given by (in the case of zero axion):

$$\begin{aligned} d\tilde{s}^2 &= e^{-\phi} ds^2\,, \quad e^{\tilde{\phi}} = e^{-\phi}\,, \\ \tilde{B} &= C_2\,, \qquad \tilde{C}_2 = -B\,, \\ \tilde{C}_4 &= C_4 + B \wedge C_2\,. \end{aligned} \tag{A.1}$$

For uplifting of a type IIA solution to eleven dimensions we use the following relations:

$$\begin{aligned} \frac{ds^2_{11}}{\ell_p^2} &= e^{-2\phi/3}\frac{ds^2_{IIA}}{\alpha'} + e^{4\phi/3}\left(\frac{dx^{11}}{R} - \frac{C_1}{\sqrt{\alpha'}}\right)^2\,, \\ \frac{A_3}{\ell_p^3} &= \frac{C_3}{(\alpha')^{3/2}} + \frac{dx^{11}}{R} \wedge \frac{B_2}{\alpha'}\,, \end{aligned} \tag{A.2}$$

where x^{11} has radius R and ℓ_p is the eleven-dimensional Planck length, which are given by $R = g\sqrt{\alpha'}$ and $\ell_p = g^{1/3}\sqrt{\alpha'}$, where g is the asymptotic value of the dilaton.

For T-duality between IIA and IIB solutions we use the Buscher rules [50] (in the string frame)

$$\begin{aligned}
\tilde{g}_{yy} &= \frac{1}{g_{yy}}\,, & \tilde{g}_{\mu\nu} &= g_{\mu\nu} - \frac{g_{\mu y}g_{\nu y} - B_{\mu y}B_{\nu y}}{g_{yy}}\,,\\
\tilde{B}_{\mu y} &= \frac{g_{\mu y}}{g_{yy}}\,, & \tilde{B}_{\mu\nu} &= B_{\mu\nu} - \frac{B_{\mu y}g_{\nu y} - g_{\mu y}B_{\nu y}}{g_{yy}}\,,\\
\tilde{g}_{\mu y} &= \frac{B_{\mu y}}{g_{yy}}\,, & e^{2\tilde{\phi}} &= \frac{e^{2\phi}}{g_{yy}}\,,
\end{aligned} \tag{A.3}$$

where y denotes the Killing coordinate with respect to which the T-dualization is applied, while μ,ν denote any coordinate direction other then y. The RR p-form fields transforms as [51]:

$$\begin{aligned}
\tilde{C}_{(p)\mu\cdots\nu\rho y} &= C_{(p-1)\mu\cdots\nu\rho} - (p-1)\frac{C_{(p-1)[\mu\cdots\nu|y|}g_{\rho]y}}{g_{yy}}\,,\\
\tilde{C}_{(p)\mu\cdots\nu\rho\sigma} &= C_{(p+1)\mu\cdots\nu\rho\sigma y} + pC_{(p-1)[\mu\cdots\nu\rho}B_{\sigma]y} + p(p-1)\frac{C_{(p-1)[\mu\cdots\nu|y|}B_{\rho|y|}g_{\sigma]y}}{g_{yy}}\,.
\end{aligned} \tag{A.4}$$

B. 'Flat space scaling' an example

In this appendix we give an example, which shows the equivalence between the 'super gravity dual' approach used in this paper (see also e.g. [9, 30, 33]) to noncommutative theories and the 'flat space scaling' approach (see e.g. [3, 11, 15, 23]). With the 'flat space scaling' approach we mean that the noncommutative theory is obtained as a limit of string/M-theory, in flat space. As an example, we will deform the M5-brane with a light-like three form A. The decoupling limit in this case is given by:

$$\begin{aligned}
&\epsilon \to 0\,, \qquad \ell_{\rm p}^2 = \ell_{\rm m}^2\epsilon\,, \qquad A_{+15} = -A_{+34} = -\epsilon^{-3/2}\,,\\
g_{++} &= -\epsilon^{-3}\,, \qquad g_{-+} = g_{+-} = 1\,, \quad x^{\pm} = \frac{1}{\sqrt{2}}(x^2 \pm x^0)\,,\\
g_{ab} &= \delta_{ab}\,, \qquad a,b = 1,3,4,5\,, \qquad g_{mn} = \epsilon^3\delta_{mn}\,, \qquad m,n = 6,\ldots,10\,.
\end{aligned} \tag{B.1}$$

This gives the following (fixed) open membrane metric and generalized noncommutativity parameter:

$$G^{\rm OM}_{\mu\nu} = \eta_{\mu\nu}\,, \qquad \Theta^{-15}_{\rm OM} = -\Theta^{-34}_{\rm OM} = \ell_{\rm m}^3\,. \tag{B.2}$$

If we now compare (B.1) and (B.2) with the supergravity dual (2.18) and open membrane data in (2.21), we have a perfect match between the two different approaches if we identify

$$\left(\frac{\tilde{r}}{\tilde{R}}\right) = \epsilon^{-1}\,. \tag{B.3}$$

This implies that the two approaches give the same decoupling limit. Conclusion: taking the decoupling limit (B.1) above corresponds in the supergravity approach, to first obtaining the M5-M2-M2-W bound state (2.16), then taking the near horizon limit (2.17), which gives the supergravity dual (2.18), followed by taking the $(\tilde{r}/\tilde{R}) = \epsilon^{-1} \to \infty$ (decoupling) limit.

To obtain the other flat space scaling limits, which corresponds to the supergravity duals (2.3) and (2.13), are straight-forward, and we will therefore not include them here.

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